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Mechanical Invariants Are Implicated in Dynamic Touch as a Function of Their Salience in the Stimulus Flow

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The authors investigated the mechanical basis of length perception through dynamic touch using specially designed rods in which the various moments of mass distribution (mass, static moment, and rotational inertia) were varied independently. In a series of 4 experiments, exploration style and rod orientation were manipulated such that the relative salience of moments of mass distribution varied markedly. Results showed that perceived length was based on the most salient moments. The authors concluded that the notion of salience is crucial for understanding the implication of moments of mass distribution in length perception and that it should play a pivotal role in developing an encompassing theory of dynamic touch.

Keywords: haptic perception, dynamic touch, inertia tensor, static moment

Dynamic touch refers to the perception of spatial and other properties of objects by hefting and wielding them without the aid of vision (Gibson, 1966). Although still poorly understood, dynamic touch is a very common form of perception that is operative whenever one drinks a cup of coffee, carries a suitcase, hits a golf ball, and so on. Because such activities are generally performed rather proficiently without seeing the object in the hand, it is assumed that the perceiver obtains action-relevant perceptual information by interacting with the objects in question.

In the search for the perceptual information that is used in dynamic touch, the main focus has been on mechanical invariants associated with the object under exploration. It has been argued that because the haptic system relies on mechanical stimulation, the information should reside in mechanical invariants that are specific to, and hence informative about, spatial and other object properties (Fitzpatrick, Carello, & Turvey, 1994; see Michaels & Carello, 1981, for a discussion on the importance of invariants in perception; Pagano, Fitzpatrick, & Turvey, 1993).

In dynamic touch, information is obtained from the dynamics of the object under exploration. Because an object's dynamics is fully determined by its moments of mass distribution, these mechanical invariants are likely to be related to the perceptual information used. The *zeroth* moment of mass distribution is the mass (m) of the object. The *first* moment of mass distribution is the static moment (M). Considering a rod held at one of its ends, the condition that was adopted in this study, we define M as the product of the rod's mass and the distance of its center of mass to

its endpoint. Finally, the *second* moment of mass distribution is the moment of inertia (I). If one considers a 2-D situation in which a rod rotates around one of its endpoints, I can be conceived as the product of the rod's mass and the squared distance of the rod's center of mass to its endpoint, multiplied by a constant. I is a dynamical invariant that describes the resistance of the rod against angular acceleration. In 3-D, the resistance against angular acceleration of a rod is rather complicated. It is described by the *inertia tensor* (I_{ij}): a symmetrical 3×3 matrix with six independent values. The greatest resistance will be experienced when wielding a rod around an axis perpendicular to its longitudinal axis. This resistance is determined by the *first eigenvalue* of the inertia tensor (I_1). The smallest resistance, determined by the *third eigenvalue* of the inertia tensor (I_3), is associated with rotating a rod around its longitudinal axis (for a more thorough treatment of the inertia tensor, see, e.g., Kingma, van de Langenberg, & Beek, 2004; Turvey, 1996).

In the search for the informational basis of dynamic touch, I_{ij} has often been put forward as an invariant of special importance. Specifically, I_1 and I_3 have been claimed to form the basis of length perception (Carello, Fitzpatrick, Flascher, & Turvey, 1998; Fitzpatrick et al., 1994; Stroop, Turvey, Fitzpatrick, & Carello, 2000; Turvey, Burton, Amazeen, Butwill, & Carello, 1998), a task that has been studied extensively in dynamic touch. However, Kingma, Beek, and van Dieën (2002) have demonstrated that the identification of I_1 and I_3 as the informational basis of length perception suffered from confounding covariation between candidate mechanical invariants. Taking the moments of mass distribution as their starting point, they showed that their own results on length perception, as well as those of previous studies that identified I_{ij} as the mechanical basis of length perception (i.e., Carello, Santana, & Burton, 1996; Pagano, Carello, & Turvey, 1996; Turvey et al., 1998), could be explained equally well by a combination of m and M .

To resolve the problem of confounding covariation, we (Kingma et al., 2004) introduced a novel experimental method. We designed rods in which m , M , I_1 , and I_3 varied independently relative to a single reference rod, rendering the interpretation of results unprob-

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lematic. Besides finding evidence for the implication of multiple mechanical invariants in the perception of rod length, we found that the information used is constrained differentially by mechanical invariants in holding compared with welding. Specifically, we found that in the perception of rod length by welding around a horizontal orientation, both I_1 and M were implicated. In holding a rod stationary in a horizontal orientation, length perception was related only to M .

In Kingma et al. (2004), we explained this differential implication of mechanical invariants by considering the mechanical differences between holding and welding. We argued on mechanical grounds that the signal-to-noise ratio of I_1 , in contrast to that of M , would probably be very low in conditions of horizontal static holding owing to a lack of angular accelerations. This could explain the sole implication of M in length perception by static holding. In welding, the signal-to-noise ratio of I_1 is likely to be more favorable, which could explain the combined implication of M and I_1 in horizontal welding. In the remainder of this article, we refer to the signal-to-noise ratio with the term *salience*, a term commonly used in (visual) perception research to describe the most conspicuous features in the stimulus array (see, e.g., Gottlieb, Kusunoki, & Goldberg, 1998; Treue, 2003).

The foregoing mechanical explanation of the results of Kingma et al. (2004) is indicative of a view on the foundation of dynamic touch that differs at least in certain regards from the currently prevailing perspective. The latter perspective takes as its departure point that the perception of a particular property must be lawfully related to a single informational structure (cf. Turvey, 1990). In accordance with this view, most studies on dynamic touch were aimed at identifying a single mechanical invariant as the basis for the perception of a particular property. In contrast, the view adopted here holds that the perception of a particular property through dynamic touch can depend on multiple invariants, whose implication is dependent on their salience in the stimulus flow. This view calls for an examination of the informational basis of dynamic touch in a variety of contexts.

The mechanical context is strongly dependent on the orientation relative to the gravitational vertical in which the exploration takes place, as we will discuss in more detail below. It is therefore remarkable that this influence has received little attention in the study of dynamic touch. Although a possible effect of gravity was reported in the pioneering study of Solomon and Turvey (1988, Experiment 7), this effect was disaffirmed in a subsequent study (Solomon, Turvey, & Burton, 1989a) and has been mostly disregarded in the study of dynamic touch since then. Given our hypothesis that the implication of mechanical invariants in dynamic touch depends on their salience in the stimulus flow, we expected that the orientation to gravity would markedly influence the implication of mechanical invariants. In the present study, we examined the influence of orientation on the mechanical basis of length perception through dynamic touch using the method of independent variation of mechanical invariants introduced by Kingma et al. (2004).

To examine the effect of orientation in holding as well as welding, we asked participants to perform length perceptions by holding and welding a rod in a horizontal as well as a vertical orientation. These conditions were introduced to test explicit, mechanically motivated hypotheses with regard to the implication of mechanical invariants in dynamic touch as a function of mechanical context. Before introducing these experimental hypothe-

ses, we should mention that we calculated the mechanical invariants of interest using a coordinate system with the endpoint of the rod as its origin. This choice departs from the currently prevailing practice in the field of dynamic touch (apart from Kingma et al., 2002, 2004) to take the wrist as the reference point for calculating mechanical parameters, an approach that suffers from several conceptual drawbacks. For this reason we provide a concise description of the mechanical rationale for our choice of coordinate system in the Appendix, as well as addressing the conceptual problems associated with using a coordinate system around the wrist.

Specific hypotheses can be generated as to the implication of mechanical invariants in different conditions. Following Kingma et al. (2002), we express the torque needed at the endpoint of a weightless rod with a point-mass attached somewhere along its length in 2-D as follows:

$$\tau = I_1 \dot{\omega} - Mg \cos(\varphi), \quad (1)$$

where τ is the torque needed at the endpoint of the rod, $\dot{\omega}$ is the angular acceleration, g is the gravitational acceleration, and φ is the angle of the rod with the horizontal. When a rod is held stationary in a horizontal orientation, the term $I_1 \dot{\omega}$ is zero, and $Mg \cos(\varphi)$ is equal to Mg . The torque needed at the endpoint is then determined by M alone. Now consider a condition in which the rod is held stationary pointing vertically downward. Both terms on the right-hand side of Equation 1 will be zero, because both $\dot{\omega}$ and $\cos(-90^\circ)$ equal zero. Hence, no torque is needed at the endpoint of the rod in this condition. Evidently, muscular effort is still needed to hold the rod above the ground in a vertical orientation, albeit much less than in horizontal holding. In Figure 1, the relative magnitude of the forces involved in horizontal and vertical holding are depicted. In a horizontal orientation, the magnitude of the upward force needed to prevent the rod from linearly accelerating downward, which is solely determined by its mass, equals the difference in absolute magnitude between the two opposing forces (i.e., F_1 and F_2). One can appreciate from Figure 1A that this difference constitutes only a small portion of the total magnitude of the forces exerted in horizontal holding. In contrast, in vertical

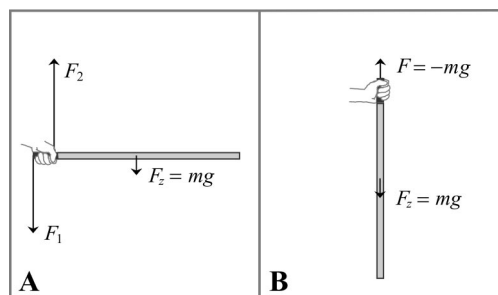


Figure 1. Simplified depiction of the forces involved in holding a rod horizontally (A) or vertically (B). Arrows represent the magnitude and direction of the forces involved in holding the rod statically. In the horizontal orientation, F_1 and F_2 represent the reaction forces to the “kick” of the rod near the base of the thumb and its “pressure” near the index finger, respectively (cf. Hoisington, 1920). Together, the forces prevent the rod from translating downward and from rotating around the endpoint. In the vertical condition, no force is needed to prevent rotation. Hence, in this condition, the rod only needs to be kept from translating downward under the influence of gravity.

holding, the force needed to prevent the rod from linearly accelerating downward is the only force that is implicated (Figure 1B). Hence, whereas the detection of m is hampered by its low salience in horizontal holding, it is the only invariant determining the exerted force in vertical holding, rendering its salience optimal. It was therefore hypothesized that m would be uniquely implicated in length perception by vertical holding.

For wielding a rod in different orientations, the predictions were not as clear-cut as for static holding. Still, differences in the implication of mechanical invariants as a consequence of a different orientation were expected. In horizontal wielding, F_1 and F_2 (shown in Figure 1) will become even greater than in horizontal holding owing to the additional force needed to impose the angular accelerations that define wielding. It was thus expected that the salience of I_1 would be high and that the salience of both m and M would be correspondingly lower. In wielding vertically, the salience of M was expected to be much lower than in horizontal wielding because the term $Mg \cos(\varphi)$ in Equation 1 would be small with φ close to -90° . As a consequence, the implication of I_1 and, possibly, m was expected to be more pronounced than in horizontal wielding because of their higher salience.

In summary, the present study was conducted to examine the theoretical position that length perception by dynamic touch is based on a multitude of mechanical invariants whose implication is a function of their salience in the stimulus flow. In Experiments 1 and 2, the effect of orientation was investigated in static holding and wielding, respectively. Experiments 3 and 4 were subsequently performed to further investigate the finding in Experiment 2 that a single invariant (M) had an opposite effect in vertical wielding compared with horizontal wielding.

Experiment 1

In the first experiment, we focused on perceiving the length of a rod by statically holding it at one of its endpoints. In an earlier study, we found that only M was implicated in the perception of rod length by pointing it forward horizontally (Kingma et al., 2004). In holding a rod that was artificially kept in a strictly

vertical orientation, Lederman, Ganeshan, and Ellis (1996) found that only the mass was implicated in length perception. In line with this result, we argued in the introduction that, on mechanical grounds, m is likely to be the dominant mechanical invariant underlying perception in statically holding a rod vertically. In the present experiment, we asked participants to statically hold a rod in two horizontal orientations and in the vertical orientation adopted by Lederman et al. (1996), but without the artificial constraint used in that study. We hypothesized that changing the orientation from horizontal to vertical would result in a considerable change of mechanical context owing to the altered gravitational influence. As a consequence, we expected a change to occur in the mechanical invariant that is exploited (i.e., M in a horizontal orientation, m in a vertical orientation). Changing the orientation in a horizontal plane was not expected to affect the implication of mechanical invariants. Finally, we hypothesized that I_1 and I_3 would not play a role in the present experiment, because angular accelerations were expected to be too small for the rod's rotational inertia to be sufficiently salient.

Method

Participants. Ten participants (5 women and 5 men; all right-handed; mean age = 23 years, $SD = 3.2$ years), who suffered from neither afflictions of the wrist nor neurological or visual impairments, participated voluntarily in the experiment after having signed a written informed consent. The selected participants were not familiar with the type of experiment or the rationale behind it.

Materials. The materials used in Kingma et al. (2004) were again used in the present experiment. They consisted of two sets of five hollow carbon fiber rods, one with 100-cm-long rods and one with 75-cm-long rods. All rods had a 10.2-cm handle at one end. The outer radius of the carbon fiber rods was 0.75 cm, and the inner radius was 0.60 cm. Two brass weights were attached to each rod. Their position on the rod, as well as their dimensions, were chosen in such a way that only one parameter from the set [I_1 , I_3 , M , and m] varied with respect to a reference rod (Table 1; see Figure 1 of Kingma et al., 2004, for an illustration of this principle of independent variation).

Each rod set thus consisted of one reference rod and four experimental rods, in each of which only one mechanical invariant was varied relative to

Table 1
Mechanical Properties of the Rods Used in Experiments 1 and 2

Rod	Length (m)	M (kg · m · 10 ¹)	I_1 (kg · m ² · 10 ²)	m (kg)	I_3 (kg · m ² · 10 ⁴)
100-cm rods					
Reference	1.00	2.11	12.40	0.43	0.89
M (−22.6%)	1.00	1.63	12.40	0.43	0.89
I_1 (+44.7%)	1.00	2.11	17.94	0.43	0.89
m (+39.6%)	1.00	2.11	12.40	0.61	0.89
I_3 (+90.3%)	1.00	2.11	12.40	0.43	1.69
75-cm rods					
Reference	0.75	1.10	4.97	0.31	0.88
M (−20.2%)	0.75	0.88	4.97	0.31	0.88
I_1 (+38.7%)	0.75	1.10	6.89	0.31	0.88
m (+40.2%)	0.75	1.10	4.97	0.44	0.88
I_3 (−57.2%)	0.75	1.10	4.97	0.31	0.38

Note. Percentages in the first column represent variation relative to the reference rod.

Numbers in boldface indicate variation relative to the reference rod. I_1 = first eigenvalue of the inertia tensor; I_3 = third eigenvalue of the inertia tensor; m = mass; M = static moment.

the reference rod. Accordingly, in the analysis of the data, we performed paired *t* tests as planned comparisons between perceived length of experimental rods and reference rod to examine the effect of each mechanical invariant separately.

Procedure. Each participant was asked to make judgments of rod length while statically holding rods in the right hand. The rods were kept out of view by an opaque curtain. This was done in three conditions: a forward condition, in which the rod was held in a horizontal orientation pointing forward relative to the participant; a sideward condition, with the rod in a horizontal orientation pointing sideward (i.e., to the left) relative to the participant; and a downward condition, with the rod in a vertical orientation pointing downward (see Figure 2). The conditions were blocked and separated into three sessions that were performed on separate days. Each session consisted of 80 trials, divided into eight blocks of 10 trials in which the rods were judged in varying random order. Each session had a short break (approximately 3 min) after 20 and 60 trials and a longer break (approximately 10 min) after 40 trials. The participant was free to take an additional break whenever he or she needed one. The six possible session orders were distributed over participants in a way that minimized the chance of an order effect.

In each condition, the participant was standing upright on a 40-cm-high plateau, which elevated his or her hand to a height of at least 150 cm above the floor (see Figure 2). The participant's right arm extended through a hole in the curtain that was adjustable in size and height. The right lower arm was positioned on a horizontal armrest with the hand extending just over the armrest's front edge.

In each trial, the experimenter handed the rod to the participant, holding it just distally from the grip with one hand and near the endpoint with the other. Subsequently, the experimenter carefully transferred the support of the rod to the participant's right hand, avoiding unnecessary rod movement during this act. The participant was asked to hold the rod motionless at all

times. After having held each rod for as long as needed to obtain a length perception, the participant indicated the perceived length of the rod by moving a rectangular surface (30×40 cm) along a rail using his or her left hand (see Figure 2). The participant was instructed to move the surface to a position where, if the surface were to extend through the screen, the tip of the rod would just touch it when the rod was held parallel to the curtain either horizontally (in the horizontal conditions) or vertically (in the downward condition). The experimenter recorded the length estimates from a measuring tape that was attached to the moving surface out of view of the participant. After each trial, the experimenter saw to it that the participant returned the surface to the same starting position corresponding to a rod length of zero.

Results and Discussion

A repeated measures analysis of variance (ANOVA) with the within-subject factors orientation (three levels: forward, sideward, and downward holding), set (two levels: 100-cm rods and 75-cm rods), rod (five levels: a reference rod, variation of I_1 , variation of M , variation of m , and variation of I_3), and repetition (eight levels) was performed. The ANOVA revealed significant main effects of orientation, $F(2, 18) = 6.3, p = .009, \eta_p^2 = .41$; set, $F(1, 9) = 35.5, p < .001, \eta_p^2 = .80$; and rod, $F(4, 36) = 20.5, p < .001, \eta_p^2 = .70$. In addition, a significant interaction effect was found between orientation and rod, $F(8, 72) = 13.8, p < .001, \eta_p^2 = .61$; orientation and set, $F(2, 18) = 10.1, p = .001, \eta_p^2 = .53$; and orientation, set, and rod, $F(8, 72) = 3.7, p = .001, \eta_p^2 = .29$. For the 100-cm rod set, mean perceived lengths were 90.7 cm, 106.7 cm, and 87.7 cm for forward, sideward, and downward holding, respectively. For the 75-cm rod set, they were 67.4 cm, 80.8 cm, and 74.0 cm, respectively. Perceived length was greatest in the sideward condition for both rod sets. From the regression coefficients in Experiments 1 and 3 of Solomon and Turvey (1988), it can be deduced that perceived length was also substantially greater in sideward wielding (Experiment 3) compared with forward wielding (Experiment 1). Thus, although the cause of the effect is unclear, it seems systematic.

Because the main focus of the present experiment was the effect of orientation on the implication of different mechanical invariants, rather than the effect of orientation as such, we subsequently performed paired *t* tests between perceived length of experimental rods and reference rod for each orientation and rod set. Note that only one mechanical invariant varied in each experimental rod, so that, in effect, we examined the effect of each mechanical invariant separately. In forward holding (Figure 3, left column), the effects of m , I_1 , and I_3 were nonsignificant in both rod sets (all $ps > .093$). Only variation of M had a significant effect on perceived length. In the 100-cm rod set, a 22.6% decrease of M resulted in a 10.1% decrease in perceived length, $t(9) = -4.6, p = .001, \eta^2 = .70$. In the 75-cm rod set, the effect of M was somewhat less pronounced, $t(9) = -3.0, p = .016, \eta^2 = .49$. The rod with a 20.2% smaller M than the reference rod was perceived as being 8.3% shorter on average.

The results for sideward holding (Figure 3, middle column) were similar to those for forward holding: The effects of m and I_1 were nonsignificant in both rod sets (all $ps > .338$). The effect of I_3 was nonsignificant in the 75-cm rod set ($p = .426$) and small (i.e., $< 2\%$) but significant in the 100-cm rod set, $t(9) = 2.9, p = .017, \eta^2 = .48$. As in forward holding, only variation of M had a marked effect on perceived length in sideward holding. In the 100-cm rod set, the effect of M was highly significant, $t(9) =$

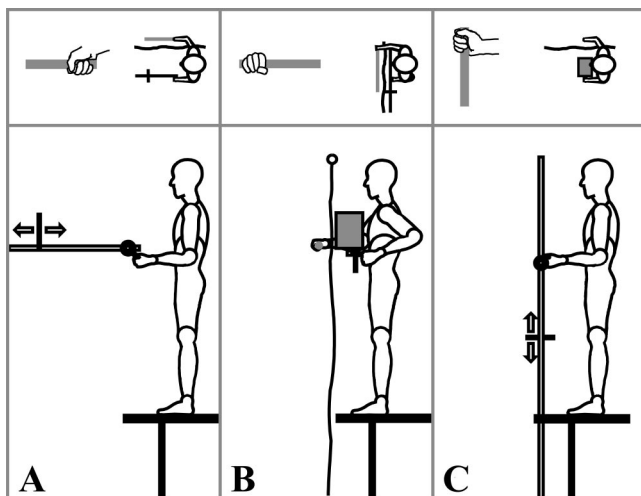


Figure 2. Setup of Experiment 1. Each participant judged the length of unseen rods that were held statically in a forward (A), sideward (B), and downward (C) orientation. In the forward condition, the participant stood on the left side of an opaque curtain. The right arm extended through a hole in the curtain, and the rod was held horizontally and parallel to the curtain. In the sideward condition, the participant was facing the curtain. Again, the rod was held horizontally and parallel to the curtain, now extending sideward relative to the participant. In the downward condition, the participant stood on the left side of the curtain, as in the forward condition. The rod was held parallel to the curtain with its distal tip pointing downward. The plateau on which the participant stood ensured that he or she did not perceive the rod to extend below the floor of the laboratory in the downward condition.

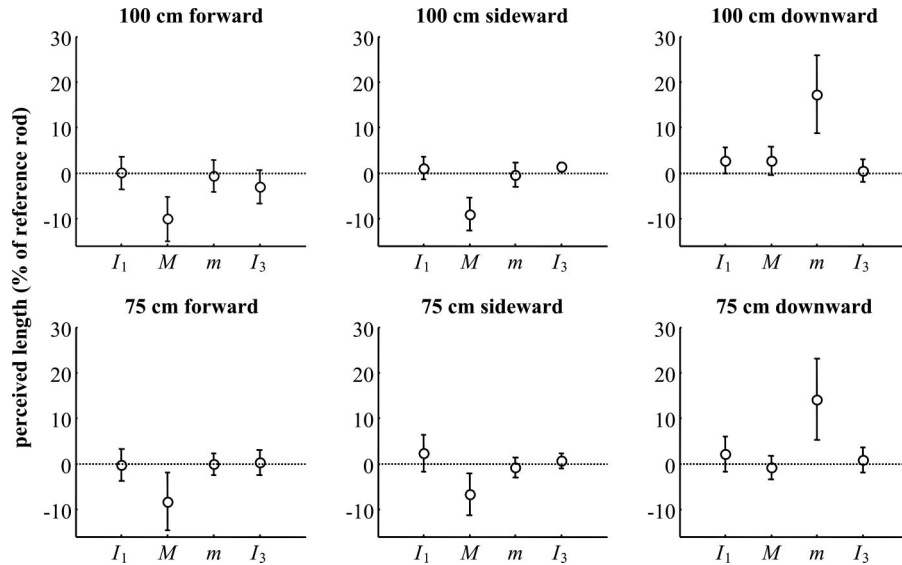


Figure 3. Results of Experiment 1. Mean perceived lengths of the rods with variation in only I_1 , M , m , and I_3 relative to the mean perceived length of the reference rod in percentages. The error bars indicate the 95% confidence interval of the difference between each experimental rod and the reference rod, as given by the planned comparisons. Separate plots are shown for the three orientations (i.e., forward, sideward, and downward) and the two rod sets (i.e., 100 cm and 75 cm). I_1 = first eigenvalue of the inertia tensor; I_3 = third eigenvalue of the inertia tensor; m = mass; M = static moment.

$-5.7, p < .001, \eta^2 = .78$. A 22.6% decrease of M resulted in a 9% decrease in perceived length. Although less pronounced, the effect of M was also significant in the 75-cm rod set, $t(9) = -3.3, p = .010, \eta^2 = .55$. On average, the 75-cm rod with a 20.2% smaller M than the reference rod was perceived as being 6.7% shorter.

The results for downward holding were very different from those for forward and sideward holding, as can be appreciated from Figure 3. Variation of M , I_1 , or I_3 was not found to affect perceived length in downward holding, although the effects of I_1 and M approached significance in the 100-cm rods ($p = .065$ and $.093$, respectively; all other $ps > .247$). Perceived length seemed to depend only on the mass of the rod, which is consistent with Lederman et al. (1996). In the 100-cm rod set, the rod with a 39.6% greater mass relative to the reference rod was perceived as being significantly longer (by 17.2%) than the reference rod, $t(9) = 4.6, p = .001, \eta^2 = .70$. In the 75-cm rod set, the rod with a 40.2% greater mass than the reference rod was perceived as being significantly longer (by 14.2%) than the reference rod, $t(9) = 3.6, p = .006, \eta^2 = .59$.

In sum, the results showed that I_1 is not implicated in length perception by static holding, which confirms the findings of Kingma et al. (2004). Furthermore, we found that the implication of mechanical invariants is a function of orientation. Whereas M seems to be the main invariant determining length perception in statically holding a rod horizontally (i.e., in a forward and a sideward orientation), m seems to be most important in vertical (i.e., downward) holding. This conclusion holds for both rod sets, that is, over a considerable range of lengths. Our prediction, which was based on a consideration of the salience of the different mechanical invariants in statically holding a rod in different orientations, thus proved to be correct.

The finding that only M was implicated in static holding, although being in line with the results of Kingma et al. (2004), seems

incompatible with the results of Experiment 2 of Stroop et al. (2000). While controlling for m and M , they found different perceived lengths in static holding conditions when I_{ij} was varied. As in Kingma et al. (2004), however, the number of repetitions was larger in the present experiment than in Experiment 2 of Stroop et al. (2000), as was the percentage of variation of I_1 . In Kingma et al. (2004), in which we used the same two rod sets, we argued that because the rods in Stroop et al. had an I_1 that was smaller than that of the rods used in Kingma et al. by a factor of approximately 5, angular accelerations due to involuntary variations in muscular torque were likely to have been much greater in their experiment than in ours. Moreover, we argued that the difference could be due to unwanted rod movement occurring at the moment each rod is transferred from experimenter to participant. We still consider it most likely that the contrasting results can be explained by one, or both, of these factors.

The present results also have important implications for the force–torque model considered by Lederman et al. (1996), as well as the static moment–torque model proposed by Carello, Fitzpatrick, Domaniewicz, Chan, and Turvey (1992). Both models were suggested to underlie static holding irrespective of the mechanical context of exploration. The implication of M in horizontal but not in vertical static holding, as found in the present experiment, is inconsistent with the static moment–torque model proposed by Carello et al. (1992). Lederman et al. (1996), in their Experiment 2, found a similar result and proposed the force–torque model as a possible alternative. However, the implication of mass, an invariant specific to *force* as defined by Lederman et al. (1996), in vertical but not in horizontal holding contradicts the force–torque model. Hence, neither the static moment–torque model nor the force–torque model can explain the present results. Note that both the study of Lederman et al. (1996) and that of Carello et al. (1992) were aimed at finding a basis for length perception by static

holding that is irrespective of mechanical context (although Lederman et al., in their general discussion, did consider the possibility that the implication of mechanical parameters is context dependent). The present results strongly suggest that this aim is unrealistic and that a single mechanical basis cannot account for length perception by static holding in both a horizontal and a vertical orientation.

Experiment 2

In a static situation, an analysis of the contribution of the different candidate mechanical invariants to the stimulus flow is not very complicated. However, when the length of a rod is perceived by wielding it, the contribution of the different invariants to the muscular force and tissue deformation is less straightforward. Wielding a rod at its endpoint in a gravitational field will cause all three moments of mass distribution to enter the equations of motion and, thus, contribute to the stimulus flow to the perceiver. Nevertheless, we hypothesized in the introduction that the implication of the different mechanical invariants would be dependent on the orientation with respect to the gravitational field (i.e., horizontal vs. vertical), as was clearly the case in the first experiment. This hypothesis was tested in the present experiment.

Method

As in Experiment 1, 10 participants (6 women and 4 men; all right-handed; mean age = 25 years, $SD = 2.6$ years), who suffered from neither afflictions of the wrist nor neurological or visual impairments, participated voluntarily in the experiment after having signed a written informed consent. The selected participants were not familiar with the type of experiment or the rationale behind it.

The same materials were used as in Experiment 1. Hence, again, we performed paired t tests as planned comparisons between perceived length of experimental rods and reference rod in the analysis of the data. As we chose to focus exclusively on the effect of a change of orientation relative to the gravitational force vector, we used only two of the orientations adopted in Experiment 1: the sideward orientation and the downward orientation. In contrast to Experiment 1, participants were now instructed to wield the rods (around the wrist) around a horizontal (sideward) or a vertical (downward) orientation roughly in a plane parallel to the curtain. Apart from the exclusion of a second horizontal orientation (i.e., with the rod pointing forward) and the instruction to wield the rods around the wrist rather than to hold them stationary, the procedure was the same as in Experiment 1. Thus, each of the 10 participants performed a total of 160 trials, divided into two sessions of 80 trials (i.e., one session for each orientation) that were performed on separate days. The order of the sessions was counterbalanced across participants.

Results and Discussion

As in Experiment 1, we performed a repeated measures ANOVA with the within-subject factors orientation (two levels: sideward and downward wielding), set (two levels: 100-cm and 75-cm rods), rod (five levels: a reference rod, variation of I_1 , variation of M , variation of m , and variation of I_3), and repetition (eight levels). The ANOVA revealed a significant main effect of set, $F(1, 9) = 31.4$, $p < .001$, $\eta_p^2 = .78$, and rod, $F(4, 36) = 16.5$, $p < .001$, $\eta_p^2 = .65$, while the main effect of orientation came close to significance, $F(1, 9) = 4.7$, $p = .058$, $\eta_p^2 = .34$. The only significant two-way interaction was that between orientation and rod, $F(4, 36) = 10.4$, $p < .001$, $\eta_p^2 = .54$. Finally, the three-way

interaction between orientation, set, and rod was significant, $F(4, 36) = 2.7$, $p < .05$, $\eta_p^2 = .23$. For the 100-cm rod set, mean perceived lengths were 85.4 cm and 77.0 cm for sideward and downward wielding, respectively. For the 75-cm rod set, they were 63.9 cm and 53.8 cm, respectively. Thus, as in Experiment 1 and in accordance with the regression coefficients presented by Solomon and Turvey (1988), perceived length tended to be greater in the sideward wielding condition, although here, in contrast to Experiment 1, the main effect of orientation did not reach significance.

As in Experiment 1, we subsequently performed paired t tests to test the effects of the mechanical invariants on perceived length in both orientations. In sideward wielding (Figure 4, left column), the 100-cm rod with a 22.6% smaller M relative to its reference rod was perceived as significantly shorter (by 8.5%) than the 100-cm reference rod, $t(9) = -3.3$, $p = .009$, $\eta^2 = .55$. Variation of I_1 and I_3 did not significantly affect perceived length in the 100-cm rods (both $ps > .20$). Mass had a small but significant effect, $t(9) = -2.8$, $p = .022$, $\eta^2 = .46$. In the 75-cm rods, only I_1 was found to affect perceived length significantly, $t(9) = 2.9$, $p = .018$, $\eta^2 = .48$. The rod with a 38.7% greater I_1 than the reference rod was perceived as being 6.4% longer than the reference rod. Static moment only showed a trend ($p = .066$), and we found no significant effects of I_3 and m (both $ps > .59$).

The absence of a significant effect of I_1 in horizontally wielding the 100-cm rods deviates from results of Kingma et al. (2004), where, using the same materials, we did find a significant effect of I_1 in horizontal wielding. The only difference between the two experiments is that wielding was forward in Kingma et al. (2004)

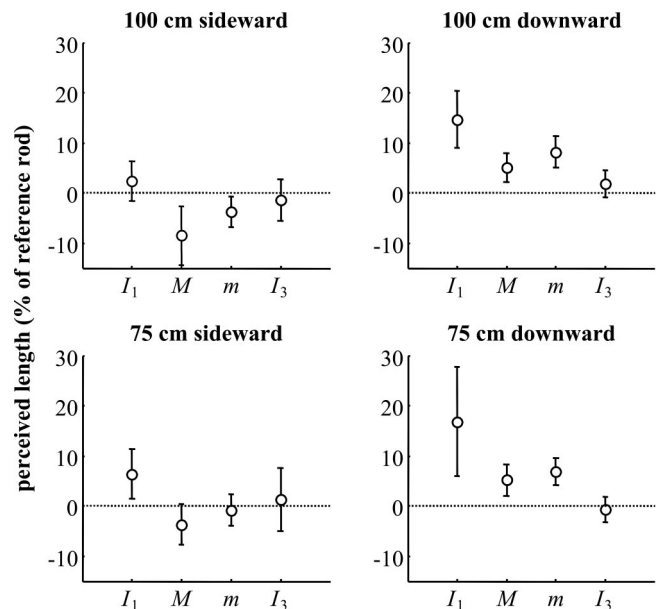


Figure 4. Results of Experiment 2. Mean perceived lengths of the rods with variation in only I_1 , M , m , and I_3 relative to the mean perceived length of the reference rod in percentages. The error bars indicate the 95% confidence interval of the difference between each experimental rod and the reference rod, as given by the planned comparisons. Separate plots are shown for the two orientations (i.e., sideward and downward) and the two rod sets (i.e., 100 cm and 75 cm). I_1 = first eigenvalue of the inertia tensor; I_3 = third eigenvalue of the inertia tensor; m = mass; M = static moment.

and sideward in the present experiment. However, an independent t test comparing the effect of I_1 in both experiments failed to reach significance, $t(18) = -1.5$, $p = .15$, $\eta^2 = .11$, implying that the discrepancy between the two experiments was not as large as suggested by the qualitative difference in statistical outcome (i.e., significant vs. nonsignificant).

As in Experiment 1, the results for vertical welding were very different from those for horizontal welding. In downward welding 100-cm rods (Figure 4, left column), significant effects of I_1 , M , and m on length perception were found, $t(9) = 5.8$, $p < .001$, $\eta^2 = .79$ for I_1 ; $t(9) = 4.0$, $p = .003$, $\eta^2 = .64$ for M ; $t(9) = 5.8$, $p < .001$, $\eta^2 = .79$ for m . All three rods were perceived as longer than the reference rod (by 14.6% for I_1 ; by 5.1% for M ; by 8.2% for m). No significant effect of I_3 was found ($p = .160$). The results for the 75-cm rod set were similar: I_1 , M , and m significantly affected length perception, $t(9) = 3.4$, $p = .007$, $\eta^2 = .58$ for I_1 ; $t(9) = 3.8$, $p = .004$, $\eta^2 = .61$ for M ; $t(9) = 5.6$, $p < .001$, $\eta^2 = .78$ for m . Again, all three rods were perceived as longer than the reference rod (by 16.8% for I_1 ; by 5.2% for M ; by 6.9% for m). No significant effect of I_3 was found ($p = .566$).

Overall, the effects were stronger in downward than in sideward welding. In particular, the effect of I_1 was magnified, as was expected on the basis of the alleged difference in salience of I_1 between sideward and downward welding. The most striking difference between sideward and downward welding, however, was that the effect of M , and to a lesser extent that of m , was opposite in sideward welding and downward welding (see Figure 4). For M , this was true for both rod sets, with the proviso that the effect of M was only a trend in sideward welding the 75-cm rods. In downward welding, the rod with smaller M was perceived as longer, implying a negative correlation between M and perceived length. The effect of m also reversed in downward welding relative to sideward welding, albeit only in the 100-cm rods. This pattern of results suggests that dynamic touch might be constrained by a combination of mechanical invariants, perhaps constituting a higher order invariant. Whereas the reversal in the effect of mass is difficult to accommodate in terms of a possible higher order invariant, such a higher order invariant involving static moment is readily identified when the oscillatory dynamics of rod welding are considered, namely the period of the rod considered as a compound pendulum. After all, this period (T) is defined as

$$T = 2\pi \sqrt{\frac{I_1}{Mg}} \quad (2)$$

and is thus constrained by the ratio of I_1 over M . In homogeneous rods, this ratio increases with increasing rod length because I_1 increases more rapidly with increasing length (i.e., with length cubed) than M (i.e., with length squared). As a consequence, the period changes linearly with the length, implying that it is specific to the length of a homogeneous rod. Decreasing M while keeping I_1 constant, as was done in the present experiment, implies an increase of the ratio of I_1 over M and thus of the period of the pendulum, indicating a longer homogeneous rod. If the abovementioned mechanical quantity indeed underlies the effects of I_1 and M in downward welding, this would identify the *period* of the rod considered as a pendulum as a relevant mechanical invariant, which is of particular theoretical significance for perception in view of its one-to-one relation with the length of a homogeneous rod. The ratio of I_1 over M was considered before, in a study by

Carello, Thuot, Anderson, and Turvey (1999), as a possible basis for perceiving the so-called "sweet spot" of a rod, but it was discarded in favor of the inertia tensor. In Experiment 3, we performed a test of the hypothesis that the ratio of I_1 over M underlies the effects of both invariants in downward welding.

Experiment 3

If the ratio of I_1 over M indeed underlies the effects of I_1 and M in downward welding, then varying them to the same degree would not affect perceived length in downward welding owing to a constant ratio. This was investigated in the present experiment by having participants perceive the length of rods in which either I_1 or M varied independently and rods in which I_1 and M covaried, rendering a constant ratio of I_1 over M (i.e., a constant period).

Method

Participants. Ten participants (6 women and 4 men, of whom 2 were left-handed; mean age = 24 years, $SD = 2.6$ years), who suffered from neither afflictions of the wrist nor neurological or visual impairments, participated voluntarily in this experiment after having signed a written informed consent. The selected participants were not familiar with the type of experiment or the rationale behind it.

Materials. Two sets of seven rods were used: a 75-cm rod set and a 50-cm rod set. All rods were made of carbon fiber and had an outer radius of 1.0 cm and an inner radius of 0.85 cm. Each rod had a grip of 10.2 cm. Each set consisted of six experimental rods in which I_1 and M were manipulated separately and in combination relative to a common reference rod (see Table 2). In the present rod sets, mechanical invariants were increased as well as decreased with respect to this reference rod. I_3 and m were the same in all rods within a set. In the data analysis, we performed paired t tests as planned comparisons between perceived length of experimental rods and reference rod to examine the effect of separately varying M and I_1 and that of varying them in combination.

For the variation of I_1 and M separately, the principle of independent variation used in Experiments 1 and 2 was again adopted in designing the present rod sets. In contrast to Experiments 1 and 2, however, the weights attached to the rods consisted solely of (lead) cylinders that were placed inside the hollow rods. Rings were not needed because m and I_3 were not varied. Below, we provide a brief explanation of the way in which equal variation of I_1 and M was achieved.

Consider a rod with one of two equal point-masses placed at its distal tip and the other placed at its proximal tip. Both I_1 and M of the proximal point-mass are zero. Therefore, when part of the proximal mass is displaced to the distal endpoint, I_1 and M will increase to the same degree. Similarly, I_1 and M will decrease to the same degree when part of the distal point-mass is displaced to the proximal endpoint. Because it is physically impossible to place a weight exactly at the proximal endpoint of a rod, the values of I_1 and M of the proximal weight, although very small, will cause the variation of I_1 and M to slightly differ in the two situations sketched above. A subsequent small displacement of the distal weight was used to correct for this small difference.

Procedure. The procedure was largely the same as in Experiment 2. Each participant was asked to make judgments of rod length while welding unseen rods in the right hand. This was done in a horizontal (forward) condition and a vertical (downward) condition. The conditions were blocked and separated into two sessions of 84 trials. The order of the sessions was counterbalanced across participants. Each session consisted of two blocks corresponding to the two rod sets. Each block of 42 trials was divided into six blocks of 7 trials in which the seven rods were judged in varying random order.

In each trial, participants were instructed to weld the rods around the wrist roughly in a plane parallel to the curtain, as in the previous experi-

Table 2
Mechanical Properties of the Rods Used in Experiment 3

Rod	Length (m)	M (kg · m · 10 ¹)	I_1 (kg · m ² · 10 ²)	m (kg)	I_3 (kg · m ² · 10 ⁴)
75-cm rods					
Reference	0.75	1.25	6.69	0.47	0.22
M (−19.4%)	0.75	1.01	6.69	0.47	0.22
M (+19.4%)	0.75	1.50	6.69	0.47	0.22
I_1 (−19.4%)	0.75	1.25	5.39	0.47	0.22
I_1 (+31.3%)	0.75	1.25	8.78	0.47	0.22
$M \& I_1$ (−30.0%)	0.75	0.88	4.67	0.47	0.22
$M \& I_1$ (+29.8%)	0.75	1.63	8.68	0.47	0.22
50-cm rods					
Reference	0.50	0.56	1.98	0.32	0.15
M (−19.4%)	0.50	0.45	1.98	0.32	0.15
M (+19.4%)	0.50	0.66	1.98	0.32	0.15
I_1 (−19.4%)	0.50	0.56	1.60	0.32	0.15
I_1 (+31.3%)	0.50	0.56	2.60	0.32	0.15
$M \& I_1$ (−30.0%)	0.50	0.39	1.39	0.32	0.15
$M \& I_1$ (+29.8%)	0.50	0.72	2.57	0.32	0.15

Note. Percentages in the first column represent variation relative to the reference rod. Numbers in boldface indicate variation relative to the reference rod. I_1 = first eigenvalue of the inertia tensor; I_3 = third eigenvalue of the inertia tensor; m = mass; M = static moment.

ments. A 10-min break was taken after 42 trials, and participants were free to take an extra break whenever they needed one.

Results and Discussion

A repeated measures ANOVA with the within-subject factors orientation (two levels: horizontal and vertical welding), set (two levels: 75 cm and 50 cm), rod (seven levels; see Table 2), and repetition (six levels) was performed. The ANOVA revealed significant main effects of set, $F(1, 9) = 122.7$, $p < .001$, $\eta_p^2 = .93$, and rod, $F(6, 54) = 53.2$, $p < .001$, $\eta_p^2 = .86$. The 75-cm rods were perceived as longer than the 50-cm rods (74.4 cm vs. 49.1 cm), and perceived length differed between rods within a set. In addition, a significant interaction between rod and set, $F(6, 54) = 2.8$, $p < .05$, $\eta_p^2 = .24$, and between orientation and rod, $F(6, 54) = 20.6$, $p < .001$, $\eta_p^2 = .70$, was found. For the 75-cm rods, mean perceived lengths were 70.8 cm and 78.0 cm for forward and downward welding, respectively. For the 50-cm rods they were 43.6 cm and 54.6 cm, respectively.

The results of subsequent paired t tests between mean perceived length of the reference rod and the experimental rods are presented in Table 3. In the horizontal condition (Figure 5; left column), the effects of varying M , I_1 , and both M and I_1 were significant in both rod sets. Varying both I_1 and M had a larger effect on perceived length than varying either of them separately, as can be appreciated from Figure 5. Note, though, that in order to correctly interpret the magnitude of variation in the dependent variable (i.e., perceived length), one should take into account the relative variations of I_1 and M in the experimental rods as shown in Table 3 (first column). A 19% decrease of only I_1 and a 19% decrease of only M resulted in a decrease of perceived length of 5% and 9%, respectively, whereas a 30% decrease of both I_1 and M was associated with a 19% decrease of perceived length. Increasing only I_1 with 31% and only M with 19% was associated with a 5% increase of perceived length in both instances, whereas a 30% increase of both I_1 and M

implied a 14% increase of perceived length. Similar results were found for the 50-cm rods. With the same relative variations as in the 75-cm rods, a separate decrease of I_1 and M was associated with a 7% decrease of perceived length in both instances, whereas a decrease of I_1 and M together implied a 21% decrease of perceived length. A separate increase of I_1 and M was associated with a 4% and 7% increase of perceived length, respectively, whereas an increase of I_1 and M together implied a 17% increase of perceived length. Although these results seem to suggest that the effect of manipulating I_1 and M in combination exceeds the sum of their separate, isodirectional effects, this is not the case. When the percentage change in the different manipulations is taken into account, it becomes apparent that the effect of manipulating both I_1 and M (while keeping the period constant) was similar in magnitude to that of the sum of the separate, isodirectional manipulations of I_1 and M . We thus concluded that the period did not play a role in our horizontal condition.

In the vertical condition (Figure 5, right column), the effect of decreasing M in both rod sets and the effects of increasing M and increasing both I_1 and M in the 75-cm rod set were not significant at the .05 level. All other effects were significant. In contrast to the horizontal condition, the effect of varying I_1 and M in combination was about equal in magnitude to the effect of varying I_1 alone. Another contrast with the horizontal condition was the negative effect of M in the 50-cm rod set. When M was increased with 19%, perceived length decreased with 4%, on average. This negative effect of M is consistent with the results of Experiment 2.

From the finding that covariation of I_1 and M (with their ratio remaining constant) affected perceived length in downward welding, we concluded that the ratio of I_1 over M , representing the period of the rod considered as a pendulum, cannot be regarded as the sole invariant explaining the effects of M and I_1 in the vertical condition. In fact, the effects can be explained for the most part by a separate effect of I_1 . The hypothesis that the period constitutes

Table 3
Planned Comparisons Between Perceived Length of Experimental Rods and Reference Rods in Experiment 3

Rod	Length (m)	Sideward welding				Downward welding			
		<i>df</i>	<i>t</i>	η^2	<i>p</i>	<i>df</i>	<i>t</i>	η^2	<i>p</i>
75-cm rods									
<i>M</i> (−19.4%)	0.75	9	−4.2	.66	.002	9	−0.7	.05	.514
<i>M</i> (+19.4%)	0.75	9	2.7	.45	.024	9	−1.7	.25	.115
<i>I</i> ₁ (−19.4%)	0.75	9	−4.0	.64	.003	9	−3.9	.63	.004
<i>I</i> ₁ (+31.3%)	0.75	9	2.3	.37	.048	9	3.3	.55	.009
<i>M</i> & <i>I</i> ₁ (−30.0%)	0.75	9	−9.0	.90	<.001	9	−4.2	.66	.002
<i>M</i> & <i>I</i> ₁ (+29.8%)	0.75	9	6.2	.81	<.001	9	1.6	.22	.150
50-cm rods									
<i>M</i> (−19.4%)	0.50	9	−3.4	.56	.008	9	0.7	.05	.516
<i>M</i> (+19.4%)	0.50	9	3.7	.60	.005	9	−2.9	.47	.019
<i>I</i> ₁ (−19.4%)	0.50	9	−3.3	.55	.009	9	−4.2	.67	.002
<i>I</i> ₁ (+31.3%)	0.50	9	2.1	.34	.061	9	7.9	.87	<.001
<i>M</i> & <i>I</i> ₁ (−30.0%)	0.50	9	−7.2	.85	<.001	9	−4.1	.65	.003
<i>M</i> & <i>I</i> ₁ (+29.8%)	0.50	9	8.4	.89	<.001	9	4.8	.72	.001

Note. Percentages in the first column represent variation relative to the reference rod. *I*₁ = first eigenvalue of the inertia tensor; *M* = static moment.

the only relevant parameter in downward welding thus has to be rejected. However, the negative effect of *M* in the 50-cm rods is consistent with the results of Experiment 2 and does suggest that, apart from *I*₁, the rod's period plays a role in downward welding. Moreover, the present results do not rule out a separate, positive effect of *M* in downward welding, similar to that in horizontal welding. When only *M* is varied, the effect of *M* alone would counteract the effect of the ratio of *I*₁ over *M*. Regarding the

information constrained by *M* alone and the information constrained by the ratio of *I*₁ over *M* as “conflicting” could explain the finding that the effect of *M* is either absent or slightly negative in vertical welding. Note that an increase of *M* can be picked up both as a decreased period (corresponding to a shorter rod) and as an increased static torque (corresponding to a longer rod). In Experiment 4, we separated these possibly counteracting effects by manipulating the task constraints in vertical welding.

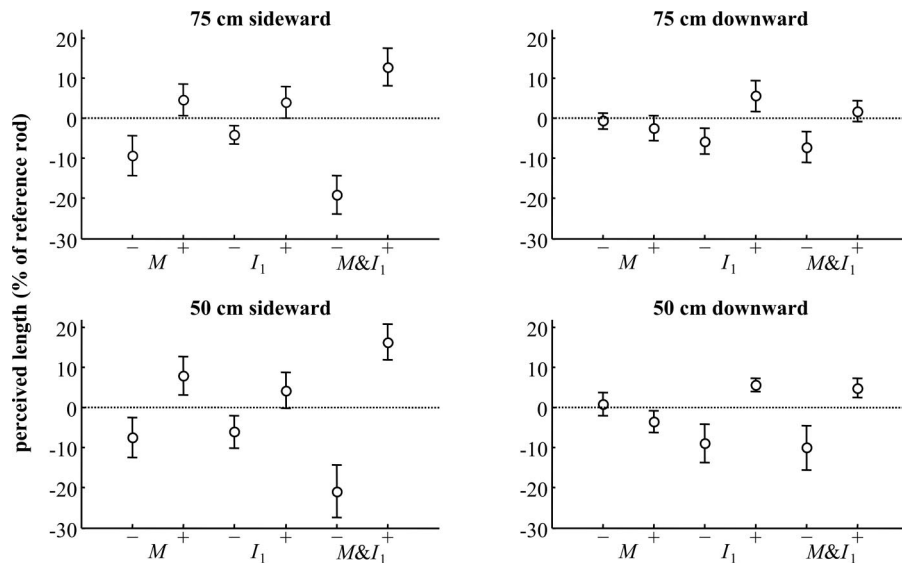


Figure 5. Results of Experiment 3. Mean perceived lengths of the rods with variation in only *I*₁, only *M*, and a combination of *I*₁ and *M* relative to the joint reference rod. The error bars indicate the 95% confidence interval of the difference between each experimental rod and the reference rod, as given by the planned comparisons. Separate plots are shown for the two orientations (i.e., horizontal and vertical welding) and the two rod sets (i.e., 75 cm and 50 cm). *I*₁ = first eigenvalue of the inertia tensor; *M* = static moment.

Experiment 4

The present experiment was aimed at determining whether the perception of rod length in downward welding indeed involves two counteracting sources of information, as we suggested above. Specifically, we hypothesized that an increased M could be picked up both as a higher static torque, corresponding to a longer rod, and as a shorter *period* of the rod considered as a pendulum, corresponding to a shorter rod. To separate these two alleged effects, we introduced different constraints on the exploration such that one of the two contradictory sources of information would be more salient than the other and therefore more likely to be picked up.

We adopted “fast” and “slow” welding conditions. In the slow condition, welding occurred at a frequency well below the eigenfrequency. We hypothesized that in such conditions dynamic properties such as the period would be largely obscured. Given a constant mass, perceived length would then be a positive function of M , because a larger M implies a larger static torque, corresponding to a longer homogeneous rod. In the fast welding condition, the effect of M on static torque was expected to be largely obscured, because oscillation was continuously forced and the rod was never held statically in a nonvertical orientation. Because the rod was forced to oscillate above its eigenfrequency, we hypothesized that the rod’s period, a dynamic property, would be more salient in such conditions. This would imply a negative relation between M and perceived length (see Equation 2).

Method

Participants. Ten participants (6 women and 4 men, all right-handed; mean age = 22 years, $SD = 3.9$ years), who suffered from neither afflictions of the wrist nor neurological or visual impairments, participated voluntarily in this experiment after having signed a written informed consent. The selected participants were not familiar with the type of experiment or the rationale behind it.

Materials. Three pairs of rods were used in the present experiment, with variation of only M in each pair. The first pair consisted of the 75-cm rods of Experiments 1 and 2 (see Table 1); the second pair consisted of the 75-cm rods of Experiment 3 with positive and negative variation of M relative to the reference rod (see Table 2); the third pair consisted of 50-cm rods in which m was 0.32 kg, I_1 was $2.7 \cdot 10^{-2} \text{ kg} \cdot \text{m}^2$, I_3 was $4.6 \cdot 10^{-5} \text{ kg} \cdot \text{m}^2$ in both rods, and M was $5.9 \cdot 10^{-2}$ and $8.2 \cdot 10^{-2} \text{ kg} \cdot \text{m}$, respectively. The variations of M in the three rod pairs (relative to the rod with the largest M) were -22.6% , -32.7% , and -28.3% , respectively. We performed planned comparisons to test the differences in perceived length between rods within each pair.

Procedure. Each participant was asked to judge the length of unseen rods that were welded pointing downward. Fast and slow welding conditions were adopted. The experiment was divided into two series of 36 trials, corresponding to the two welding conditions. Each series consisted of three blocks corresponding to the three rod pairs, in which each pair was judged six times. The order in which the rods of a pair were presented was randomized.

Participants were told that the main goal of the experiment was to test the accuracy of their length estimates in different conditions. The experimental task was essentially the same as in the downward welding condition in Experiment 2. In the slow welding condition, participants were told to slowly explore different orientations of the rod that slightly deviated from the vertical and to grip the rod firmly. They were told further that it was not allowed to adopt orientations that deviated more than 30° from the vertical orientation, and to move the rod roughly in a vertical plane parallel to the curtain. This resulted in a welding frequency well below the rod’s

eigenfrequency, which was 0.8 Hz on average. In the fast welding condition, participants were instructed to wield the rods fast and parallel to the curtain and to grip the rod firmly. They were told further that it was not allowed to hold the rod stationary in a nonvertical orientation, and that the rod could not deviate more than 30° from the vertical during the exploration. This instruction resulted in the rod oscillating well above its eigenfrequency.

A break of 10 min was taken after the first series of 36 trials, and participants were allowed to take an extra break whenever they desired one. Although each block of 12 judgments in reality consisted of only two rods, all participants were very surprised to hear that they had been performing 72 trials with only six rods.

Results and Discussion

A repeated measures ANOVA with the within-subject factors rod (two levels: low and high M), set (three levels: 75-cm, 50-cm, and 25-cm rod pairs), exploration (two levels: slow and fast welding), and repetition (six levels) was performed. The only significant main effect was that of set, $F(2, 18) = 32.2$, $p < .001$, $\eta_p^2 = .78$, and the only significant interaction was that between rod and exploration, $F(1, 9) = 14.9$, $p = .004$, $\eta_p^2 = .62$. In the slow welding condition, mean perceived lengths were 68.4 cm and 62.0 cm for the rods with high and low M , respectively. By contrast, in the fast welding conditions, they were 61.8 cm and 67.1 cm, respectively. Hence, the slow and fast welding conditions indeed resulted in the expected inversion of the effect of M .

Subsequently, we performed paired t tests between the rods within a pair in the two exploration conditions separately (see Figure 6). In the slow welding condition (Figure 6, upper panel), rods in which M was greater were perceived as being significantly

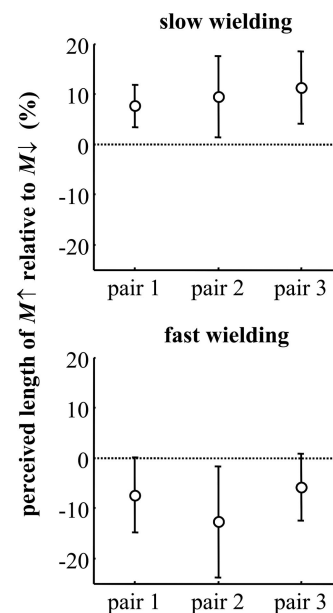


Figure 6. Results of Experiment 4. Difference in mean perceived length between the rod with the smallest M ($M \downarrow$) and the rod with the largest M ($M \uparrow$) in each pair in percentages. The error bars indicate the 95% confidence interval of these differences, as given by the planned comparisons. A positive difference implies that $M \uparrow$ was perceived to be longer. Separate plots are shown for the two welding conditions (i.e., slow welding and fast welding).

longer, $t(9) = -4.1$, $p = .003$, $\eta^2 = .65$ for the first pair; $t(9) = -2.7$, $p = .026$, $\eta^2 = .44$ for the second pair; $t(9) = -3.5$, $p = .007$, $\eta^2 = .58$ for the third pair. In the fast wielding condition (Figure 6, lower panel), mean perceived length of the rods within a set in which M was greater was consistently smaller than that of the other rods, which is in contrast to the slow wielding condition. The difference in perceived length within the first rod pair came very close to reaching significance, $t(9) = 2.3$, $p = .051$, $\eta^2 = .36$, and was significant in the second rod pair, $t(9) = 2.6$, $p = .028$, $\eta^2 = .43$. Within the third rod pair, the difference was not significant, although a trend was still observed, $t(9) = 2.0$, $p = .076$, $\eta^2 = .31$.

The results of Experiment 4 strongly supported the hypothesis that participants are sensitive to the ratio of I_1 over M (i.e., the period) in downward wielding. In fast wielding, where the period of the rod was expected to be particularly salient, the effect of M was indeed found to be negative. In slow wielding, where we expected that the effect of varying M on the period of the rod would be obscured, the effect of M was positive, as in horizontal wielding (see Experiments 2 and 3).

General Discussion

In the present study, we tested the verity of a novel view on the mechanical basis of dynamic touch, which started to take shape in Kingma et al. (2004). There, we found that in length perception, distinct mechanical invariants were implicated in wielding compared with static holding. Notably, I_1 was not implicated in holding a rod stationary, whereas it was when the rod was wielded. We concluded that the absence of angular accelerations in static holding prevented the pickup of I_1 . In the present study, we anticipated that the differential implication of mechanical invariants as found in Kingma et al. (2004) would form no exception. In contrast to recent accounts of dynamic touch (see, e.g., Cooper, Carello, & Turvey, 1999, 2000; Pagano et al., 1996; Turvey, 1996), we anticipated that the pickup of rotational inertia is not a hallmark property of length perception through dynamic touch. The results of Kingma et al. (2004) highlighted instead the flexible exploitation of mechanical invariants. Rather than pursuing an account of dynamic touch based on a single invariant, we therefore examined the flexible, context- and task-dependent use of multiple mechanical invariants.

Our results confirmed that the mechanical basis of length perception through dynamic touch was different for static holding than for wielding. Moreover, they showed that the implication of mechanical invariants is a function of the orientation of the rod relative to the gravitational vertical and of exploration style. Hence, our results constitute strong support for the point of view that perceivers are flexible in exploiting mechanical invariants in dynamic touch.

It is theoretically important to note that the role of distinct mechanical invariants in the different conditions adopted in the present study implies a role for gravity in dynamic touch. In a zero-gravity environment, horizontal exploration and vertical exploration would not constitute different mechanical contexts. Hence, the context-dependent implication of mechanical invariants depends to a large degree on the presence of a constant gravitational field. Although the pioneering studies of Solomon and colleagues (Solomon & Turvey, 1988; Solomon, Turvey, & Burton, 1989b) postulated the inertia tensor as the unique basis for

length perception through dynamic touch, they already hinted at a possible influence of gravity. Solomon et al. (1989a) subsequently addressed this possibility explicitly. In their Experiment 2, they had participants perform length perceptions in a horizontal and a vertical orientation and found a significant main effect of orientation on perceived length, which they explained by suggesting that gravity could have made a scalar contribution to length perception. In their general discussion, however, their results were interpreted as support for the claim that the perception of rod length through wielding is not affected by a change of coordinates. An effect of orientation on haptically perceived length was also reported by Carello et al. (1992). Again, however, the finding was given little attention in the discussion of their results. Since Carello et al. (1992), the present study is the first to examine the effect of orientation on the haptic perception of rod length. The results indicate that the role of gravity has been underestimated in the study of dynamic touch.

We submit that the basis for the context-dependent implication of mechanical invariants, whether induced by gravity or otherwise, is the salience of the candidate invariants in the different contexts. Recall that the implication of specific invariants in the different conditions adopted in the present study could be readily deduced from a consideration of their salience in the stimulus flow, which supports our view. An exception appeared to be our finding in Experiment 2 that an increase of M was associated with the perception of a longer rod when the rod was wielded horizontally, whereas it was associated with a shorter rod when that rod was wielded pointing downward. We originally assumed that any effect of M would be based on the torque involved in holding or wielding a rod at an angle to the gravitational vertical. According to this assumption, the effect of M on perceived length, if any, should always be positive. It can be shown, however, that an increased static moment decreases the rod's period when it behaves like a pendulum under the influence of gravity (see Equation 2). Equation 2 shows that the sensitivity of the perceptual system to the period of the rod implies an invariant composed of the ratio of I_1 over M .

In Experiments 3 and 4, we investigated whether this ratio indeed underlay perception in downward wielding. The results of Experiment 3 showed that the period was not the only invariant that was implicated. In fact, the results of Experiment 3 could be explained for the most part by a separate effect of I_1 . However, a role of the period could not be ruled out, because, as in Experiment 2, a negative effect of M was found in downward wielding. We recognized a possibility that would render the small negative effect of M of considerable importance for the theoretical understanding of the informational basis of dynamic touch. That is, when only M was varied in downward wielding, the negative effect of the ratio of I_1 over M could have been counteracted by a positive effect of M based on torque. We expected that the pickup of an increased M as an increased torque would be facilitated by slow exploratory movement around a vertical orientation, whereas a faster exploration, in which the rod's movements are more likely to resemble the oscillatory behavior of a pendulum, would favor the pickup of the ratio of I_1 over M . To separate these two effects, we examined both exploratory styles in our fourth and final experiment. As expected, we found that the direction of the effect of M reversed, now upon a change of exploration style rather than orientation. Slow wielding rendered the effect of M positive, whereas fast wielding resulted in a negative relation between M and perceived length. The

findings of Experiment 4, in conjunction with the negative relationship between M and perceived length found in Experiments 2 and 3, strongly suggest the implication of the ratio of I_1 over M in downward wielding and thus provide evidence for the existence of yet another mechanical invariant in length perception through dynamic touch.

The multitude of mechanical invariants found to be implicated in length perception does not fare well with the prevailing view in the study of dynamic touch that experiments should be aimed at identifying a unique mechanical basis. By adopting this latter view, one would most likely attempt to explain the context-dependent implication of a multitude of mechanical invariants in the perception of rod length by supposing the existence of a single, yet to be identified invariant that is somehow constrained by distinct moments of mass distribution in different mechanical conditions (see Cutting, 1991, for related arguments against multiple sources of information). Indeed, this argument was advanced by Burton and Turvey (1990), in response to their finding that in static conditions, the first moment of mass distribution underlay length perception, whereas in dynamic conditions, the second moment was found to be implicated (Solomon & Turvey, 1988; Solomon et al., 1989b). This argument, however, is untenable in light of our finding in Experiments 2, 3, and 4 that a single mechanical invariant (M) can have an opposite effect on perception in different mechanical contexts, as we argue next.

Note that dynamic touch is based on the relationship between the forces exerted by the perceiver and the resulting object motion. That is, dynamic touch is based on the object's *dynamics*. Therefore, if dynamic touch is based on invariants in the stimulus flow, these invariants must be available in the dynamics. Because m , M , and I_{ij} constitute invariants that fully determine the dynamics of an object, it follows that it is possible to express any invariant in dynamic touch in terms of these moments of mass distribution. If we were to assume the existence of a single invariant as a basis for a particular perception, we should thus be able to describe it in terms of m , M , and I_{ij} . That is, there should be a lawful relationship between the invariant and the moments of mass distribution implicated in it, irrespective of experimental conditions. In Experiments 2, 3, and 4, however, we found no such lawful relationship between M and perceived length across the exploration conditions imposed; when M was decreased relative to the reference rod, perceived length decreased in horizontal wielding and increased in vertical wielding. These results provide evidence for many-to-one mappings of information to perception, rather than one-to-one mappings.

Cutting advocated the implication of multiple sources of information in his concept of *directed perception* (Cutting, 1986). On this view, a percept is constructed through the selection and integration of multiple sources of information available in the stimulus flow. Our results seem to support this view in that they provide evidence for the multitude of informational structures implicated in perception. However, in our view, many-to-one mappings of information to perception do not imply that a percept is constructed through a process of selection and integration. First, we do not hold that the perceiver (or the perceiver's brain) selects among different informational structures. Instead, we hold that the relative salience of an informational structure, rather than an act of selection on the part of the perceiver, determines whether it is implicated in the perception. Second, in our view, if one finds that a particular perception is based on an integrated combination of

invariants, this integrated combination constitutes yet another invariant structure in the stimulus flow. The implicated invariants need not be detected separately and subsequently integrated. A complex combination of multiple invariants need not render the corresponding information more difficult to pick up, as was noted by Runeson (1977) when he introduced his concept of smart perceptual mechanisms. From the finding that perception is based on a particular combination of invariants, it must be concluded that the perceptual system is sensitive to information that can be described as such. We consider it wrong to assume that the *mechanism* through which this information is picked up is revealed by its description in terms of a combination of invariants, which seems to be implied by the concept of directed perception (Cutting, 1986, 1991). For example, we do not imply that in length perception by downward wielding, I_1 and M are picked up and subsequently divided in the perceiver's brain. Rather, we hypothesize that the perceiver is sensitive to the period of the rod considered as a pendulum.

Thus, the present results are compatible with the concept of perception as the detection of information. Moreover, the notion of a flexible use of mechanical invariants on the basis of their salience in the stimulus flow seems to be an expansion of this concept, rather than being contradictory to it.

We conclude with a discussion of some interesting implications of the present results for future experimental studies on dynamic touch. Specifically, we anticipate that the notion of salience will be important in providing insight into perceptual learning and individual differences in dynamic touch. We noticed in the present series of experiments that large differences exist in the exploration styles of different participants. Such differences could result in differences in the informational basis of perception between participants, as they are likely to affect the salience of the mechanical invariants that define this basis. Relatedly, it is to be expected that in the course of learning to perceive through dynamic touch, the dynamics of exploration will change so as to optimize the salience of the most useful, most informative invariants for a given perceptual task. Not only would this highlight the ecological notion of perception as an active process of information pickup rather than the passive reception of stimuli, it would also underscore the significance of the notion of salience in understanding the basis of differences in invariant use between individuals, as well as within individuals in the course of perceptual learning. In research on visual perception, such differences in invariant use have been repeatedly reported (Bingham, McConnell, & Muchisky, 2001; Jacobs, Michaels, & Runeson, 2000; Michaels & De Vries, 1998). In the study of dynamic touch, however, the topic has received little attention. A study by Wagman, Shockley, Riley, and Turvey (2001) and a recent study by Withagen and Michaels (2005) are the only studies to have addressed individual differences and perceptual learning in dynamic touch. They also reported that the mechanical invariants underlying perception vary between individuals and in the course of learning. It would be interesting to see whether such differences are accompanied by an exploration style that implies a high salience of the invariants that are implicated. Combining the issues of individual differences and learning by exploring the differences in exploration style and invariant use between novices and experts (e.g., fencers, golfers) also seems to provide a fruitful path toward further unraveling the mechanical basis of dynamic touch.

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(Appendix follows)

Appendix

Considerations Regarding the Appropriate Coordinate System in Dynamic Touch

In the past 17 years, many studies have been conducted to uncover the mechanical parameters that underlie dynamic touch. In nearly all of these studies, the center of rotation in the wrist was adopted as the origin of the coordinate system relative to which mechanical parameters should be considered. This reference point was assumed to be imposed on a perceiver by the laws of rotational dynamics. In the following, we argue that this assumption is incorrect and that mechanical invariants are considered more appropriately relative to a coordinate system originating at the perceiver–object interface in the grip.

A mechanical analysis reveals what is required for a perceiver to detect moments of mass distribution relative to a rotation center in the wrist. Consider a rod being wielded solely around the wrist (w) and held at its end, as shown in Figure A1. Relative to a principal coordinate system, fixed in the hand–rod system and originating at w , the torque is generally expressed as

$$\tau_{\text{tot},w} = I_{\text{hr},w} \cdot \dot{\omega} + \omega \times (I_{\text{hr},w} \cdot \omega), \quad (\text{A1})$$

where $\tau_{\text{tot},w}$ denotes the total torque relative to w , $I_{\text{hr},w}$ is the diagonalized inertia tensor of the hand–rod system relative to w , and ω and $\dot{\omega}$ denote angular velocity and acceleration of the principal, noninertial coordinate system. This equation is mathematically equivalent to Euler's dynamical equations of motion. The variable $\tau_{\text{tot},w}$ represents the sum of a gravitational torque and a torque applied by the perceiver. To express only the latter ($\tau_{\text{perc},w}$), the gravitational torque should be taken to the right-hand side of the equation. We get

$$\tau_{\text{perc},w} = I_{\text{hr},w} \cdot \dot{\omega} + \omega \times (I_{\text{hr},w} \cdot \omega) - m_{\text{hr}} \cdot \mathbf{wy} \times (R \cdot \mathbf{g}'), \quad (\text{A2})$$

where the gravitational torque is written as $m_{\text{hr}} \cdot \mathbf{wy} \times (R \cdot \mathbf{g}')$, with m_{hr} representing the mass of rod and hand, \mathbf{wy} the vector from wrist to the combined center of mass of rod and hand, R a variable 3×3 orientation matrix specifying the orientation of the principal coordinate system relative to the inertial reference frame, and \mathbf{g}' the constant gravitational acceleration vector in the inertial reference frame. The right-hand side of Equation A2 now contains all variables and invariants contributing to the torque exerted at the stationary wrist. The first two terms contain the invariant inertia tensor, and the third term contains the invariant static moment, written as $m_{\text{hr}} \cdot \mathbf{wy}$. To extract these invariants, a perceiver's sensitivity to the self-applied torque as well as the variables on the right side of Equation A2 is required.

It is important to appreciate that Equations A1 and A2 hold only for rotations around a fixed point. That is, they hold only for the special case in which the center of rotation in the wrist is (or at least may be assumed to be) stationary. Pagano, Fitzpatrick, and Turvey (1993) compared this restricted condition with a more natural condition where wielding was allowed to occur around shoulder, elbow, and wrist simultaneously.

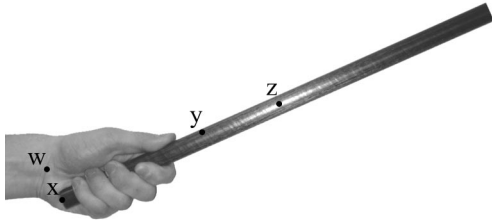


Figure A1. Representation of a rod held at its end, with w representing the center of rotation in the wrist, x the rod endpoint, y the center of mass of hand and rod combined, and z the center of mass of the rod alone.

Clearly, the wrist is not stationary in the latter free-wielding condition. Of note, however, they found that length perception was independent of the joint(s) used in wielding, and they concluded that the same coordinate system must have been adopted in both the restricted and the free-wielding condition. They declared the wrist as the origin of this common coordinate system. To accommodate the findings of Pagano, Fitzpatrick, and Turvey (1993) while maintaining a coordinate system originating at the wrist, Equation A2 needs to be modified into

$$\tau_{\text{perc},w} = I_{\text{hr},w} \cdot \dot{\omega} + \omega \times (I_{\text{hr},w} \cdot \omega) - m_{\text{hr}} \cdot \mathbf{wy} \times (R \cdot \mathbf{g}' - \mathbf{a}_w), \quad (\text{A3})$$

where \mathbf{a}_w is the wrist's linear acceleration contributing to $\tau_{\text{perc},w}$. Thus, in addition to the variables in Equation A2, sensitivity to the linear acceleration of the principal coordinate system is required to extract the invariants of exploratory dynamics.

Although Equation A3 now accommodates wrist movement, there are still two problems with the assumption that the wrist is the relevant origin. First, Equation A3 holds only under the assumption that the hand–rod system is a rigid body. That is, it holds only when the rod does not move in the hand. Without the explicit instruction to grip the rod firmly, perceivers will fail to meet this requirement. Relative to the wrist, the moments of mass distribution will then be a function of time and cannot be informative about any invariant object property.

The second problem follows from the appreciation that the mechanical invariants in Equations A1 through A3 must refer to the hand–rod system, not to the rod alone. Extracting either the static moment or the inertia tensor of the rod alone from the torque around the wrist is impossible. It has to be concluded that a rod's moments of mass distribution do not constitute invariants in the pattern of torques around the wrist.

To properly study exteroception by dynamic touch, we should consider invariants that a perceiver may indeed extract in natural circumstances, where none of our experimental restrictions apply. We argue below that considering a rod's moments of mass distribution relative to a position in the grip reveals just such invariants.

Consider again the situation of Figure A1. With an equation similar to Equation A3, the torque applied around x , the rod endpoint, can be expressed as follows:

$$\tau_{\text{perc},x} = I_{\text{r},x} \cdot \dot{\omega} + \omega \times (I_{\text{r},x} \cdot \omega) - m_{\text{r}} \cdot \mathbf{xz} \times (R \cdot \mathbf{g}' - \mathbf{a}_x). \quad (\text{A4})$$

The similarity between Equations A3 and A4 is apparent. Essentially the same variables are required for extracting invariants relative to the wrist and relative to the rod endpoint in the hand. However, whereas the extraction of invariants referring to the rod alone from the torque around the wrist is impossible, because torque around the wrist is not directly applied to the rod, their extraction relative to x is a viable option, because x is situated in the grip, at the interface between hand and rod, where a perceiver applies torque directly and exclusively to the rod. Consequently, in contrast to Equation A3, Equation A4 expresses the possibility of extracting the genuinely exterospecific invariants $I_{\text{r},x}$ and $m_{\text{r}} \cdot \mathbf{xz}$, which refer exclusively to the object to be perceived. Finally, in contrast to the mechanical invariants in Equation A3, those in Equation A4 remain constant in the absence of the instruction to maintain a firm grip, when mass distribution relative to the wrist varies.

In summary, adopting a principal coordinate system originating in the grip is subject to similar requirements to those for a coordinate system originating at the wrist, while avoiding the conceptual problems associated with the latter.

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